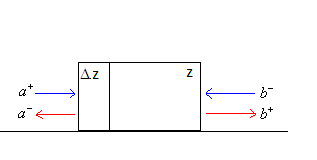
**DMPK Summary**

The failure of the RMT approach suggested there were more correlations between the eigenvalues than symmetry alone necessitated. To make further progress, a different approach was initiated. Some of the assumptions made along the way turned out to limit the applicability of the results to Q1D. Nonetheless, the approach forms the basis for attempted generalizations to higher dimensions and so we’ll start here. Consider a transfer matrix of length z.



As the length increases, we would expect its probability distribution function to change, and to eventually approach a limiting distribution in accordance with Oseledec’s theorem. We can then write down a recursive relation for the probability distribution function of the transfer matrix (see Appendix something),



where **M**ʹ is the transfer matrix corresponding to the slice Δz, **M**ʹʹ the transfer matrix of the length z and **M** the composite transfer matrix. In terms of the polar representation (see another Appendix),



We can put the λ´´, u´´, υ´´ in terms of λ, u, υ, λ´, u’, υ´, by forming **M**´´ and constructing M´´12M´´21T and M´´12T M´´21 respectively to get:





This enables us to identify u´´, υ´´†, and λ´´ as the eigenvectors and eigenvalues of the respective expressions. These equations reveal an important form for uʹʹ, υʹʹ, and λʹʹ:



At this point, it is desired to go to the continuum limit. To that end, we slice the Δz interval into smaller pieces, dz, and attempt to work out the probability distribution of the **M** variables within that slice. Absent any particular insight into the nature of the pdz(**M**), we turn to a maximum entropy approach. Other than normalization, a requirement we can impose on pdz(**M**) comes from the conductance. For the thin slice, we expect λ to be small, and so:



In the ballistic limit, we also expect g to go as g0e-dz/ℓ, where ℓ is the mean free path. Comparing the two formulas, to first order we would expect:



At this point we acknowledge no further information characterizing pdz(**M**), and so we use the maximum entropy principle.



Fixing the constants to enforce normalization, and the average, we obtain,



where c(N) is a normalization constant – note Pdz(M) would include the measure |λi – λj|β term. Note that this probability distribution predicts the following averages, which will be used later:



These isotropic averages are taken to be justified in the Q1D limit, but Tartakovsky and others have pointed out that this does not hold in actuality; nevertheless the DMPK follows. At this point we are formally done. What remains is to expand uʹʹ, υʹʹ, and λʹʹ, about u, υ, and λ respectively, out to O(dL) in δλʹʹ, δuʹʹ, and δυʹʹ. This can be done using 2nd order perturbation theory, as according to the pdz, the matrices V, and W are of order √dz. For instance,



[Note in 1D, we must use a direct Taylor series expansion of δλʹʹ about λʹ = 0, since these perturbative formulas would be singular] Then once the averages are computed, we would have our FP equation for pz(u,υ,λ). Since we’re mainly interested in the marginal probability distribution pz(λ), however, we would have to take the solution to this equation and integrate ∫dμ(u)∫dμ(υ) out the u, υ dependence. It turns out, though, that the model pdz(**M**ʹ), spares us some of this trouble, and we will be able to write down an evolution equation for pz(λ) directly. The simplifying feature that makes this possible is the isotropy of pdz(u,υ,λ), i.e., its independence of u and υ. Since it is isotropic, the distribution at all lengths, pz(u,υ,λ) will be as well. Mello proves this by induction. Assuming it to be true for pz, and taking advantage of the invariance of the matrix measure dμ(υʹ) = dμ(υ†·υʹ), which allows us to ‘divide out’ the υ from the argument of λʹʹ, we come to:



which proves that pz+dz(u,υ,λ) is also isotropic. Thus we may proceed directly to a FP equation for p(λ) itself by integrating both sides of this equation against dμ(u)dμ(υ) (and performing the trivial dμ(uʹ) integral).



Now we Taylor expand λʹʹ = λ + δλʹʹ in powers of δλʹʹ, out to order dz (which will be second order in δλʹʹ) and execute the requisite disorder averages. Dividing both sides of our expansion by dz, we obtain the FP equation for pz(λ). In terms of the probability distribution function Pz(λ) = J(λ)pz(λ), it reads:



[where the ξ = (N+1)ℓ/2 is the localization length] The fact that the probability distribution function can be written as a total derivative is consistent with probability conservation. More generally, we have:



where,



See pg. 30 of Beenaker’s review for some references on early work on 1D chain scaling equation. In same sentence he lists reviews by people who considered the possibility of a ‘strong disorder’ 1D scaling equation…

**Results**

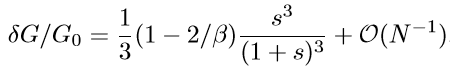
We can solve the GDMPK exactly in all cases it seems. The β = 2 case is special in that the solution can be resolved into a Slater determinant ‘wavefunctions’. In either case, the conducting and insulating regimes can be studied using a 1/N expansion (N >> s = Lz/ℓ >> 1). Some predictions from the DMPK equation are (β = 1) (from Markos):

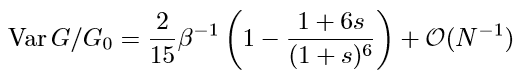


where,



Note that <g> isn’t governed by a single parameter. But this is not unexpected, since the scaling hypothesis applies only to equidimensional metals. Results for all ensembles for Lz << ξ, in powers of 1/N…note G0 is the conductance quantum.





Random comment. P. Lee says that the broadness of the insulating distribution, has to do with the fact the metal’s distribution is dominated by rare events of transparency, and that these happen when the incident electron is resonant with a localized state of the same energy situated in the middle of the sample.

In the ‘thick wire’ limit, the DMPK equation has been shown to be equivalent to the (1D) NLsM. Thick wire means N >> 1 (thick), and (Lz >> ℓ) (wire) with ratio Nℓ/Lz fixed. And 1D is reference to fact that the fields are assumed to vary only along Lz (seems like a sort of isotropy assumption which probably precludes 3D). It is noted that DMPK is still more general in that while Q1D and restricted to weak disorder (like NLsM to some extent at least), it can accommodate small N.